

## Solving $x^2 + bx + c = 0$ by Completing the Square

### Review: Perfect Square Trinomials

$$\overset{x}{x^2} + \overset{\checkmark}{8x} + \overset{4}{16}$$
$$(x + 4)^2$$

$$\overset{x}{x^2} - \overset{\checkmark}{10x} + \overset{5}{25}$$
$$(x - 5)^2$$

## Completing the Square

Complete the square to form a perfect square trinomial and then factor.

$$x^2 + 12x + \boxed{6^2} \qquad z^2 - 4z + \boxed{2^2}$$

$$(x + 6)^2$$

$$(z - 2)^2$$

$$x^2 - 100x + \boxed{50^2}$$

$$(x - 50)^2$$

## Completing the Perfect Square Trinomial

**Step 1:** Identify b (coefficient of the middle term)

**Step 2:** Find  $\left(\frac{b}{2}\right)^2$

**Step 3:** Add  $\left(\frac{b}{2}\right)^2$

**Step 4:** Factor

## Solving $x^2 + bx + c = 0$ Completing the Square

**Step 1:** Write the equation in the form

$$x^2 + bx + \boxed{\phantom{00}} = c + \boxed{\phantom{00}}$$

**Step 2:** Find  $\left(\frac{b}{2}\right)^2$  and to both sides of the equation

**Step 4:** Factor the perfect-square trinomial

**Step 5:** Take square roots of both sides

**Step 6:** Write and solve two equations

## Completing the Square

**Example: Solve by completing the square.**

1)  $x^2 + 12x + 11 = 0$

$$x^2 + 12x + \boxed{36} = -11 + \boxed{36} \quad \text{Complete the perfect square}$$

$$\sqrt{(x+6)^2} = \sqrt{25} \quad \text{Factor the Perfect Square Trinomial}$$

$$x+6 = \pm 5 \quad \text{Take Square Roots}$$

$$\begin{array}{r} x+6 \\ -6 \\ \hline x = -6 \pm 5 \end{array}$$

$$\begin{array}{l} \swarrow \quad \searrow \\ x = -6 + 5 \quad x = -6 - 5 \\ \boxed{x = -1} \quad \boxed{x = -11} \end{array}$$

## Completing the Square

Solve the following by completing the square.

$$2) x^2 - 2x = 1$$

$$x^2 - 2x + \boxed{(1)^2} = 1 + \boxed{(1)^2} \quad \text{Complete the Square}$$

$$\sqrt{(x-1)^2} = \sqrt{2} \quad \text{Factor the Perfect Square Trinomial}$$

$$x-1 = \pm\sqrt{2} \quad \text{Take Square Roots}$$

$$\begin{array}{c} +1 \quad +1 \\ x = 1 \pm \sqrt{2} \end{array}$$

$$\boxed{x = 1 + \sqrt{2}}$$

$$\boxed{x = 1 - \sqrt{2}}$$