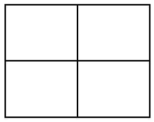
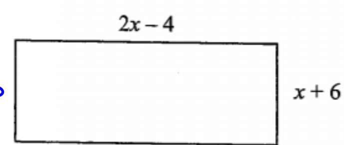
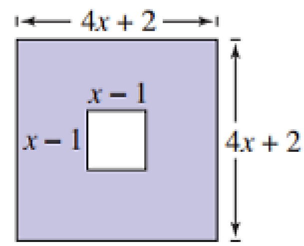
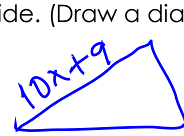
Foundations of Algebra
Unit 1: Relationships between Quantities & Expressions

Name: ______________________________
Date: ____________________ Block: _______

Unit 1 Study Guide

What you need to know & be able to do	Things to remember	Examples	
1. Identifying Parts of Algebraic Expressions	• Identify Parts of an expression Variable Constant Term Coefficient Factors	a. Identify the: $32x^2 - 8x + 4y - 9$ **Terms:** $32x^2, -8x, +4y, -9$ **Variables:** x, y **Constants:** -9 **Factors of 1st term:** $4 \cdot 8 \cdot x \cdot x$ $2 \cdot 16 \cdot x \cdot x$	b. Identify the: $24x^2 - x - 7$ **Terms:** $24x^2, -x, -7$ **Constants:** -7 **Coefficients:** $24, -1$ **Factors of 2nd term:** $-1 \cdot x$
2. Classifying Polynomials	**"First Name"** – degree #: constant x: linear x^2: quadratic x^3: cubic **"Last Name"** – number of terms 1: monomial 2: binomial 3: trinomial 4+: polynomial	Write in standard form & classify: a. $3x + 7x^2 - 5x$ $7x^2 - 2x$ quadratic binomial b. 23 constant	Write in standard form & classify: c. $2x^2 - 2x + 5 - x^2$ $1x^2 - 2x + 5$ quadratic trinomial d. $2x^3 - 4x + x^2 - 3x + 1$ $2x^3 + x^2 - 7x + 1$ cubic polynomial
3. Adding & Subtracting Polynomials	• Get rid of parentheses first • For subtraction, change the sign of everything in the parentheses after the subtracting sign • Then combine like terms	a. Simplify: $(3x^2 - 4x + 8) + (2x - 7x^2 - 5)$ $3x^2 - 4x + 8 + 2x - 7x^2 - 5$ $-4x^2 - 2x + 3$ Classification: quadratic trinomial	b. Find the sum: $(7y^2 + 2y - 3) + (2 - 4y + 5y^2)$ Classification:
		c. What is the result of: change signs $(3x^2 - 3x - 5) - (2x^2 + x - 6)$ $3x^2 - 3x - 5 - 2x^2 - x + 6$ $1x^2 - 4x + 1$ Classification: quadratic trinomial	d. Simplify: change signs $(4x^2 + 2x - 7) - (-2x^2 + 5x + 3)$ $4x^2 + 2x - 7 + 2x^2 - 5x - 3$ $6x^2 - 3x - 10$ Classification: quadratic trinomial
		e. Determine the values of m & n. $(2x^2 + 3x - 8) + (mx^2 - nx + 4) = 5x^2 - 8x - 4$ $2x^2 + 3x - 8 + mx^2 - nx + 4 = 5x^2 - 8x - 4$ $m = 3$ $n = 11$	f. Determine the values of m & n. $(mx^2 - 6x + 3) - (4x^2 + nx - 4) = -2x^2 - 3x + 7$ $mx^2 - 6x + 3 - 4x^2 - nx + 4 = -2x^2 - 3x + 7$ $m = 2$ $n = -3$

4. Multiplying Polynomials	• Set up "Area Model" • Combine like terms 	a. Find the product: $(x + 5)(x – 2)$ $\begin{array}{c\|c\|c} & x & +5 \\ \hline x & x^2 & +5x \\ \hline -2 & -2x & -10 \end{array}$ $x^2+5x-2x-10$ $\boxed{x^2+3x-10}$	b. Simplify: $(x – 5)^2$ $(x-5)(x-5)$ $\begin{array}{c\|c\|c} & x & -5 \\ \hline x & x^2 & -5x \\ \hline -5 & -5x & +25 \end{array}$ $\boxed{x^2-10x+25}$
		c. What is the product of $(2x – 4)$ and $(3x + 3)$ $(2x-4)(3x+3)$ $\begin{array}{c\|c\|c} & 2x & -4 \\ \hline 3x & 6x^2 & -12x \\ \hline +3 & +6x & -12 \end{array}$ $\boxed{6x^2-6x-12}$	d. Simplify: $(x –3)\ (x + 3)$ $\begin{array}{c\|c\|c} & x & -3 \\ \hline x & x^2 & -3x \\ \hline +3 & +3x & -9 \end{array}$ $x^2 \cancel{+3x} \cancel{-3x} - 9$ $\boxed{x^2-9}$
5. Applications of Polynomials	**Perimeter** • Fill in the missing sides • Add all sides on the exterior of the figure **Area** • Use the area formula for the respective figure • Rectangle = lw • Triangle = $\frac{bh}{2}$ • Use "Area Model" to multiply if necessary	a. Find the perimeter and area of the following figure: $x+6$  $2x-4$ Perimeter = $2x-4$, $2x-4$ → -8; $x+6$, $x+6$ → $+12$ $6x+4$ Area = $(2x-4)(x+6)$ $\begin{array}{c\|c\|c} & 2x & -4 \\ \hline x & 2x^2 & -4x \\ \hline +6 & +12x & -24 \end{array}$ $2x^2+8x-24$	b. Find the area of the shaded region.  Larger area – Smaller area $(4x+2)(4x+2) - (x-1)(x-1)$ change $(16x^2+16x+4) - (x^2-2x+1)$ $16x^2+16x+4 - x^2+2x-1$ $\boxed{15x^2+18x+3}$
		c. In 2014, the number of apples harvested at a local farm was represented by the expression $8x^2 + 2x + 3$. In 2015, the number of apples harvested was represented by the expression $6x^2 + 5x + 4$. Write a polynomial that represents the **total** number of apples harvested in 2014 and 2015, in terms of x. add $(8x^2+2x+3)+(6x^2+5x+4)$ $8x^2+2x+3+6x^2+5x+4$ $14x^2+7x+7$	d. The measure of the perimeter of a triangle is $41x + 33$. It is known that two of the sides of the triangle have measures of $18x + 12$ and $10x + 9$. Find the length of the third side. (Draw a diagram)  $18x+22$ $10x+9$ $28x+31$ $(41x+33)-(28x+31)$ $41x+33-28x-31$ $\boxed{13x-2}$

<table>
<tr>
<td rowspan="2" colspan="2">6. Simplifying Radicals
If the problem contains a perfect square:
Find the square root
The square root would be an integer
If the problem contains a number that is not a perfect square:
Use the product of two square roots
One of these roots should be a perfect square
Find the square root of the perfect square, leave the other root as is.
If the problem contains an even exponent:
Divide the exponent by 2
If the problem contains an odd exponent:
Break the problem up into 2 powers
One should have the highest even exponent
The other exponent should be 1
The sum of both exponents should be the original exponent</td>
<td>a. $\sqrt{36}$ $= 6$</td>
<td>b. $-3\sqrt{60}$
$-3\sqrt{4}\sqrt{15}$
$-3\cdot 2\sqrt{15}$
$\boxed{-6\sqrt{15}}$
60: 1 | 60, 2 | 30, 3 | 20, 4 | 15, 5 | 12, 6 | 10</td>
</tr>
<tr>
<td>c. $\sqrt{54a^4b^{10}}$
$\sqrt{54}\sqrt{a^4}\sqrt{b^{10}}$
$\sqrt{9}\sqrt{6}\cdot a^2\cdot b^5$
$3\sqrt{6}\cdot a^2\cdot b^5$
$\boxed{3a^2b^5\sqrt{6}}$
54: 1 | 54, 2 | 27, 3 | 18, 6 | 9</td>
<td>d. $3x\sqrt{16x^5y^2}$
$3x\sqrt{16}\sqrt{x^5}\sqrt{y^2}$
$3x\cdot 4\cdot\sqrt{x^4}\sqrt{x}\cdot y$
$3x\cdot 4\cdot x^2\sqrt{x}\cdot y$
$\boxed{12x^3y\sqrt{x}}$</td>
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<td rowspan="2">7. Multiplying Radicals</td>
<td rowspan="2">Remember your rules of exponents
Multiply outside numbers/variables together
Multiply inside numbers/variables together
Simplify</td>
<td>a) $5\sqrt{6}\bullet 2\sqrt{6}$
$10\sqrt{36}$
$10\cdot 6$
$\boxed{60}$</td>
<td>b) $-2\sqrt{3x}\bullet 4\sqrt{3x}$
$-8\sqrt{9x^2}$
$-8\sqrt{9}\cdot\sqrt{x^2}$
$-8\cdot 3\cdot x$
$\boxed{-24x}$</td>
</tr>
<tr>
<td>c) $2\sqrt{x^3}\bullet 2\sqrt{x^4}$
$4\sqrt{x^7}$
$4\sqrt{x^6}\sqrt{x}$
$\boxed{4x^3\sqrt{x}}$</td>
<td>d) $3\sqrt{18a^2b}\bullet 4\sqrt{3ab^3}$
$12\sqrt{54a^3b^4}$
$12\sqrt{54}\sqrt{a^3}\sqrt{b^4}$
$12\sqrt{9}\sqrt{6}\cdot\sqrt{a^2}\sqrt{a}\cdot b^2$
$12\cdot 3\sqrt{6}\cdot a\sqrt{a}\cdot b^2$
$\boxed{36ab^2\sqrt{6a}}$</td>
</tr>
<tr>
<td>8. Adding & Subtracting Radicals</td>
<td>Simplify ALL radicals first!
Then add/subtract like radicals.</td>
<td>a. $8\sqrt{7}-3\sqrt{7}$
$\boxed{5\sqrt{7}}$</td>
<td>b. $4\sqrt{6}-3\sqrt{24}$
$4\sqrt{6}-3\sqrt{4}\sqrt{6}$
$4\sqrt{6}-3\cdot 2\sqrt{6}$
$4\sqrt{6}-6\sqrt{6}$
$\boxed{-2\sqrt{6}}$</td>
</tr>
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		c. $3\sqrt{20}+2\sqrt{60}-6\sqrt{5}$ $3\sqrt{4}\sqrt{5}+2\sqrt{4}\sqrt{15}-6\sqrt{5}$ $3\cdot 2\sqrt{5}+2\cdot 2\sqrt{15}-6\sqrt{5}$ $6\sqrt{5}+4\sqrt{15}-6\sqrt{5}$ $\boxed{4\sqrt{15}}$	d. $5\sqrt{2}\left(3\sqrt{10}-2\sqrt{5}\right)$ $5\sqrt{2}(3\sqrt{10})-5\sqrt{2}(2\sqrt{5})$ $15\sqrt{20}-10\sqrt{10}$ $15\sqrt{4}\sqrt{5}-10\sqrt{10}$ $15\cdot 2\sqrt{5}-10\sqrt{10}$ $\boxed{30\sqrt{5}-10\sqrt{10}}$
		e. $\sqrt{12w}+\sqrt{27w}$ $\sqrt{12}\sqrt{w}+\sqrt{27}\sqrt{w}$ $\sqrt{4}\sqrt{3}\sqrt{w}+\sqrt{9}\sqrt{3}\sqrt{w}$ $2\sqrt{3w}+3\sqrt{3w}$ $\boxed{5\sqrt{3w}}$	f. $4\sqrt{2x}\left(3\sqrt{2x}-2\sqrt{5x^4}\right)$ $12\sqrt{4x^2}-8\sqrt{10x^5}$ $12\cdot 2\cdot x-8\sqrt{10}\sqrt{x^4}\sqrt{x}$ $24x-8\sqrt{10}\cdot x^2\sqrt{x}$ $\boxed{24x-8x^2\sqrt{10x}}$
9. Rational and Irrational Numbers	$\sqrt{3}(\sqrt{3}+2)$ $\sqrt{9}+2\sqrt{3}$ $3+2\sqrt{3}$	Classify the following as: rational or irrational. a. $\sqrt{9}$ $=3$ R b. $\sqrt{7}$ I c. $\sqrt{4}+\sqrt{9}$ R $2+3=5$ d. $\sqrt{7}+\sqrt{4}$ I e. $\sqrt{3}(\sqrt{3}+2)$ I f. $\sqrt{25}+\pi$ I	g. Explain the whether the outcome is rational or irrational $\sqrt{4}+\sqrt{16}$. (Perfect Squares) $2+4=6$ Rational h. ~~Explain~~ the outcome of $2\sqrt{2}(5+\sqrt{2})$ $2\sqrt{2}\cdot 5+2\sqrt{2}\cdot\sqrt{2}$ $10\sqrt{2}+2\sqrt{4}$ $10\sqrt{2}+2\cdot 2$ Irrational $10\sqrt{2}+4$
		i. Which sum is rational? ~~a.~~ $\sqrt{5}+2.1$ b. $\sqrt{9}+6.25$ $=3+6.25$ 9.25 ~~c.~~ $\sqrt{3}+\pi$ ~~d.~~ $\pi+12$	j. Which product is irrational? ~~a.~~ $\sqrt{6}\cdot\sqrt{6}=\sqrt{36}=6$ ~~b.~~ $\sqrt{49}\cdot\sqrt{25}$ ~~c.~~ $\sqrt{2}\cdot\sqrt{32}=\sqrt{64}=8$ d. $\sqrt{12}\cdot\sqrt{2}=\sqrt{24}$

10. Metric Conversions

King	Henry	Died	Unexpectedly	Drinking	Chocolate	Milk
k	h	d	U	d	c	m
kilo	hecto	deka	UNIT GRAM METER LITER	deci	centi	milli

When moving the decimal to the left, you are dividing by a power of 10.

When moving the decimal to the right, you are multiplying by a power of 10.

When comparing two quantities, make sure they are in the same unit before comparing (you might have to convert one of them to the other unit first).

Convert the following:

a. 12.54 km = 1254,000 cm

12.540,0,0

b. 457 mL = .00457 hL

.00457

c. 0.55 dkg = 55 dg

0.55

Compare the following: (< , >,or =) (Convert to the same units first!)

e. 7,225 cm ≃ 72.25 m

7225 cm

f. 34 g = 0.34 hg

34g

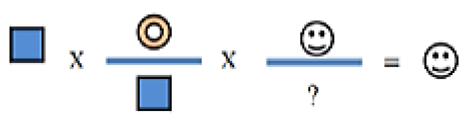
g. A recipe for shortbread cookies calls for 5 grams of butter to make 12 cookies. How many deci-grams will there be in 60 cookies?

h. A rectangle has a length of 18 meters and a width of 500 centimeters. What is the perimeter, in centimeters, of the rectangle?

11. Unit Conversions (1 & 2 Step)

Conversion Factor: $\frac{\text{what you want}}{\text{what you have}}$

Remember this activity:



If you are going from Metric to Customary or vice versa, the conversion factor will be given to you.

a. Convert 7 miles to feet.

$\frac{7\cancel{mi}}{1} \cdot \frac{5280ft}{1\cancel{mi}} = \frac{36{,}960ft}{1}$

$= \boxed{36{,}960 ft}$

$1 mi = 5{,}280 ft$

b. Convert 5 years into days.

$\frac{5\cancel{yrs}}{1} \cdot \frac{365 \text{ days}}{1\cancel{yr}} = \boxed{1{,}825 \text{ days}}$

c. How many miles will a person run during a 10 kilometer race? (1 mi = 1.6 km)

$\frac{10\cancel{km}}{1} \cdot \frac{1 mi}{1.61\cancel{km}} = \frac{10 mi}{1.61}$

$= \boxed{6.21 mi}$

d. How many gallons are in 600 quarts? 1 gal = 4 qt

$\frac{600\cancel{qt}}{1} \cdot \frac{1 gal}{4\cancel{qt}} = \frac{600 gal}{4}$

$= \boxed{150 gal}$

12. Multi-Step Dimensional Analysis

Make sure you write every single conversion factor!

Think about where you are starting and where you want to go. Create a plan that includes the necessary conversion factors.

Example: A bucket has 4.65 L of water. How many gallons of water is that (1.06 qt = 1 L).

Given: 4.65 L   Needed: gallons

Plan: L ⟶ qt ⟶ gallon

Equalities: 1.06 qt = 1 L; 1 gal = 4 qt

Set Up Problem:

$$4.65 \cancel{L} \times \frac{1.06 \cancel{qt}}{1 \cancel{L}} \times \frac{1 \text{ gal}}{4 \cancel{qt}} = 1.23 \text{ gal}$$

a. Convert 12 pints to gallons.

pt → qt → gal

$$\frac{12 \cancel{pt}}{1} \cdot \frac{1 \cancel{qt}}{2 \cancel{pt}} \cdot \frac{1 \text{ gal}}{4 \cancel{qt}}$$

$$\frac{12 \text{ gal}}{8} = \boxed{1.5 \text{ gal}}$$

m → cm → in → ft

b. Sarah ran a 10 meter race. How many feet is that? (1 in = 2.54 cm)

$$\frac{10 \cancel{m}}{1} \cdot \frac{100 \cancel{cm}}{1 \cancel{m}} \cdot \frac{1 \cancel{in}}{2.54 \cancel{cm}} \cdot \frac{1 \text{ ft}}{12 \cancel{in}}$$

$$\frac{1{,}000 \text{ ft}}{30.48} = \boxed{32.81 \text{ ft}}$$

c. A bowl of cereal weighs 60 oz. How heavy is it in kg? (1 oz = 28.3 g)

oz → g → kg

$$\frac{60 \cancel{oz}}{1} \cdot \frac{28.3 \cancel{g}}{1 \cancel{oz}} \cdot \frac{1 \text{ kg}}{1000 \cancel{g}}$$

$$\frac{1698 \text{ kg}}{1000} = \boxed{1{,}698 \text{ kg}}$$

d. John lives 4.1 miles from work (Use 1 mi = 1609 meters).
Kevin lives 2.5 kilometers from work.
Bill lives 1800 meters from work.
Jess lives 290,000 centimeters from work.
Put them in order from who lives closest to the work to who lives the farthest from work. Show your work.

13. Rate Conversions

Sometimes it is helpful to convert either the numerator or denominator first and then convert the other. If you do too much at once, your problem gets messy.

Example: Convert 66 feet per second to miles per hour.

$$\frac{66 \cancel{feet}}{1 \cancel{sec}} \cdot \frac{60 \cancel{sec}}{1 \cancel{min}} \cdot \frac{60 \cancel{min}}{1 hour} \cdot \frac{1 mile}{5280 \cancel{feet}} = 45 \; {miles}/{hour}$$

a. Convert 65 mph to feet per minute.

mi → ft

1 mi = 5280 ft

hr → min

1 hr = 60 min

$$\frac{65 \cancel{mi}}{1 \cancel{hr}} \cdot \frac{5280 \text{ ft}}{1 \cancel{mi}} \cdot \frac{1 \cancel{hr}}{60 \text{ min}}$$

$$\frac{343200 \text{ ft}}{60 \text{ min}} = \boxed{5{,}720 \text{ ft/min}}$$

b. Convert 32 feet/seconds to meters/min (1 inch = 2.54 cm).

$$\frac{32 \cancel{ft}}{1 \cancel{sec}} \cdot \frac{12 \cancel{in}}{1 \cancel{ft}} \cdot \frac{2.54 \cancel{cm}}{1 \cancel{in}} \cdot \frac{1 \text{ m}}{100 \cancel{cm}} \cdot \frac{60 \cancel{sec}}{1 \text{ min}}$$

$$= \frac{58521.6 \text{ m}}{100 \text{ min}} = \boxed{585.216 \text{ m/min}}$$

c. The average American student is in class 330 minutes/day. How many hours per school week is this (use 1 school week = 5 days)?

$$\frac{330 \cancel{min}}{1 \cancel{day}} \cdot \frac{1 \text{ hr}}{60 \cancel{min}} \cdot \frac{5 \cancel{days}}{1 \text{ school wk}} = \frac{1650 \text{ hr}}{60 \text{ sch wk}}$$

$$= \boxed{27.5 \text{ hours/school week}}$$