Algebra 1

Unit 2B/3B Notes:
Linear & Quadratic Functions

**DISCLAIMER:** We will be using this note packet for Unit 2B/3B. You will be responsible for bringing this packet to class EVERYDAY. If you lose it, you will have to print another one yourself. An electronic copy of this packet can be found on my class blog.
<table>
<thead>
<tr>
<th>Standard</th>
<th>Lesson</th>
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<tr>
<td><strong>Write expressions in equivalent forms to solve problems</strong>&lt;br&gt;MGSE9–12.A.SSE.3</td>
<td>Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.</td>
</tr>
<tr>
<td>MGSE9–12.A.SSE.3a</td>
<td>Factor any quadratic expression to reveal the zeros of the function defined by the expression.</td>
</tr>
<tr>
<td>MGSE9–12.A.SSE.3b</td>
<td>Complete the square in a quadratic expression to reveal the maximum and minimum value of the function defined by the expression.</td>
</tr>
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<td><strong>Understand the concept of a function and use function notation</strong>&lt;br&gt;MGSE9–12.F.IF.1</td>
<td>Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and f(x) is the output (an element of the range). Graphically, the graph is y = f(x).</td>
</tr>
<tr>
<td>MGSE9–12.F.IF.2</td>
<td>Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</td>
</tr>
<tr>
<td><strong>Interpret functions that arise in applications in terms of the context</strong>&lt;br&gt;MGSE9–12.F.IF.4</td>
<td>Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior.</td>
</tr>
<tr>
<td>MGSE9–12.F.IF.5</td>
<td>Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.</td>
</tr>
<tr>
<td><strong>Analyze functions using different representations</strong>&lt;br&gt;MGSE9–12.F.IF.7</td>
<td>Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.</td>
</tr>
<tr>
<td><strong>Build a function that models a relationship between two quantities</strong>&lt;br&gt;MGSE9–12.F.BF.1</td>
<td>Write a function that describes a relationship between two quantities.</td>
</tr>
<tr>
<td>MGSE9–12.F.IF.9</td>
<td>Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.</td>
</tr>
</tbody>
</table>
Unit 2B/3B: Linear and Quadratic Functions

In this unit, you will learn how to do the following:

**Unit 2B: Linear Functions**
- Learning Target #1: Creating and Evaluating Functions
- Learning Target #2: Graphs and Characteristics of Linear Functions
- Learning Target #3: Applications of Linear Functions

**Unit 3B: Quadratic Functions**
- Learning Target #4: Different Forms of Quadratic Functions and their Graphs
- Learning Target #5: Applications of Quadratic Functions

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### Unit 2B/3B Timeline

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<tr>
<td>October 7th</td>
<td>Day 1: Intro to Functions</td>
<td>8th</td>
<td>9th</td>
<td>10th  Early Release (3rd Block only)</td>
</tr>
<tr>
<td>Day 1: Intro to Functions</td>
<td>Day 2: Evaluating Functions</td>
<td>9th Day 3: Writing Linear Functions – Slopes &amp; y-Intercepts</td>
<td>10th</td>
<td>11th Day 4: Multiple Representations of Linear Functions</td>
</tr>
<tr>
<td>14th Midterm</td>
<td>15th Midterm</td>
<td>16th Midterm</td>
<td>17th</td>
<td>18th Day 7: Graphing in Intercept Form</td>
</tr>
<tr>
<td>Day 8: Converting between Forms of a Parabola</td>
<td>Day 9: Converting to Vertex Form by Completing the Square</td>
<td>16th Midterm PSAT Day</td>
<td>17th</td>
<td>21st</td>
</tr>
<tr>
<td>Day 10: Applications of the Vertex</td>
<td>Unit 2B/3B Review</td>
<td>23rd Midterm Day 10: Applications of the Vertex</td>
<td>24th</td>
<td>25th Unit 2B/3B Test</td>
</tr>
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</table>
Day 1: Introduction to Functions

Standard(s): MGSE9-12.F.IF.1 Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value.

Relation
• A relation can be represented as a: __________, __________, _________ or _______________.

Function
• A relation that maps each ______ to ______ one ___________.
• No input has more than one output (No x-values going to two different y-values)

Domain and Range
• Domain – set of all ___ values (input)
• Range – set of all ___values (output)

Determine if the following are functions. Then state the domain and range:

a. b. {(3, 4), (9, 8), (3, 7), (4, 20)}
c. {(15, -10), (10, -5), (5, 2), (10, 5), (15, 10)}

Function? Explain: Domain: Range:
Function? Explain: Domain: Range:
Function? Explain: Domain: Range:

Table:

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>20</td>
</tr>
<tr>
<td>-5</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
</tr>
</tbody>
</table>

Function? Explain: Domain: Range:
Function? Explain: Domain: Range:
Function? Explain: Domain: Range:
Vertical Line Test

- Consider all the vertical lines that could be drawn on the graph of the relation.
- If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function.

Ex. Use the Vertical Line Test to determine if the graphs of the relations are functions.

A.  

B.  

C.  

Function?  
Explain:  
Function?  
Explain:  
Function?  
Explain:

Discrete and Continuous Functions

- **Discrete function** - a function with distinct and separate values.
  
  Example: number of students at SCHS.

- **Continuous function** - a function that can take on any number within a certain interval.
  
  Example: height, age, time
State the domain and range for each graph and then tell if the graph is a function (write yes or no).

If the graph is a function, state whether it is discrete, continuous or neither.

1) Domain__________  2) Domain__________  3) Domain__________  
   Range__________    Range__________    Range__________    
   Function?__________ Function?__________ Function?__________

4) Domain__________  5) Domain__________  6) Domain__________  
   Range__________    Range__________    Range__________    
   Function?__________ Function?__________ Function?__________

7) Domain__________  8) Domain__________  9) Domain__________  
   Range__________    Range__________    Range__________    
   Function?__________ Function?__________ Function?__________
### Day 2: Function Notation

**Standard(s): MGSE9-12.F.IF.1** Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value.

- Using function notation is like replacing ____ with ____, so that we have \( f(x) = mx + b \) instead of \( y = mx + b \).
- \( f(x) \), which is read “f of x,” where \( f \) names the function.
- It shows the input (x) and output (y) pair of values of a functional relationship at the same time.

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**Evaluating Functions**

If \( f(x) = 4 - 5x \), \( g(x) = 2x^2 + 14x - 16 \), and \( p(t) = 3(2)^t - 1 \), evaluate the following using understanding of function notation.

- a. \( f(-2) \)
- b. \( g(-1) \)
- c. \( p(0) \)

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**Evaluating a Function from a Graph**

Given this graph of \( f(x) \), evaluate the following:

- a. \( f(-2) = \)
- b. \( f(0) = \)
- c. \( f(2) = \)
- d. \( f(____) = 3 \)
- e. \( f(____) = -1 \)
- f. \( f(____) = 4 \)
Applications of Evaluating Functions

Scenario 1: While visiting her grandmother, Fiona Evans found markings on the inside of a closet door showing the heights of her mother, Julia, and her mother’s brothers and sisters on their birthdays growing up. From the markings in the closet, Fiona wrote down her mother’s height each year from ages 2 to 13. Her grandmother found the measurements at birth and one year by looking in her mother’s baby book. The data is provided in the table below, with heights rounded to the nearest inch.

<table>
<thead>
<tr>
<th>Age (yrs.)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (in.)</td>
<td>21</td>
<td>30</td>
<td>35</td>
<td>39</td>
<td>43</td>
<td>46</td>
<td>48</td>
<td>51</td>
<td>53</td>
<td>55</td>
<td>59</td>
<td>62</td>
<td>64</td>
<td>64</td>
</tr>
</tbody>
</table>

a. Which variable is the independent variable? Dependent variable?

b. What is \( h(11) \) and what does this mean in context?

c. Express how tall her mother was at age 10 using function notation.

d. What is \( a \) such that \( h(a) = 53 \) and what does this mean in context?

e. What would be an appropriate domain and range for this function?

Scenario 2: You determine while walking home from school one day, you live approximately 3000 feet away from school and you can walk 5 feet every second. You determine the function \( d(t) = 3000 - 5t \) models how far, \( d \), you have left to walk after \( t \) seconds walking home.

a. What is the independent variable?

b. What is the dependent variable?

c. How far will you be from home in one MINUTE?

d. How long does it take you to be a HALF MILE from home?

e. If you live 3000 feet from school, what would be an appropriate domain and range be for this situation?
Day 3: Writing Linear Functions (Slopes and Y-intercepts)

**Standard(s):** MGSE9-12.F.BF.1 Write a function that describes a relationship between two quantities.

**Slope-Intercept Form**

*(Gives the equation of a linear function)*

\[ f(x) = mx + b \]

\[ m: \text{slope} \quad b: \text{y-intercept} \]

### Calculating Slope

<table>
<thead>
<tr>
<th>Representation</th>
<th>Formula</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Table</strong></td>
<td>( \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} )</td>
<td>where ((x_1, y_1) &amp; (x_2, y_2)) are coordinate points</td>
</tr>
</tbody>
</table>
| **Graph**     | \( m = \frac{\text{rise}}{\text{run}} \)  
\[ m = \frac{y_2 - y_1}{x_2 - x_1} \] | where \((x_1, y_1) & (x_2, y_2)\) are coordinate points |
| **Ordered Pairs** | \( m = \frac{y_2 - y_1}{x_2 - x_1} \) | \((-2, 1) \text{ and } (3, 6)\) |
Writing a Linear Equation from Graph

Find the slope and y-intercept of each graph and write the equation of the line in slope-intercept form.

A. Slope: _____  y-intercept: _____
   Equation: ___________________

B. Slope: _____  y-intercept: _____
   Equation: ___________________

Writing a Linear Equation Given 2 Points

Slope Formula

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

where \((x_1, y_1) \) & \((x_2, y_2)\) are coordinate points

Ex. Calculate the slope of two points using the slope formula.

A. \((9, 3), (19, -17)\)  
B. \((1, -19), (-2, -7)\)

How do you find the equation of the line in slope-intercept form?

- Plug in one ordered pair \((x, y)\) and the slope, \(m\) into the equation \(y = mx + b\)
- Find \(b\)
- Write in slope-intercept form \((y = mx + b)\)

What is the equation of the line in A?  
What is the equation of the line in B?
Writing a Linear Equation from a Table

Find the slope and y-intercept of each table and write the equation of the line in slope-intercept form.

A. Slope: _____  y-intercept: _____  
   Equation: ________________

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>13</td>
</tr>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>4</td>
<td>-62</td>
</tr>
<tr>
<td>10</td>
<td>-152</td>
</tr>
</tbody>
</table>

B. Slope: _____  y-intercept: _____  
   Equation: ________________

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>7</td>
<td>22</td>
</tr>
<tr>
<td>9</td>
<td>26</td>
</tr>
</tbody>
</table>

What do you do when the y-intercept cannot be found in the table?

C. Slope: _____  y-intercept: _____  
   Equation: ________________

D. Slope: _____  y-intercept: _____  
   Equation: ________________

How many pills were in the bottle to start?

How much was admission to the carnival?

<table>
<thead>
<tr>
<th>Days Passed</th>
<th>Vitamins Remaining in Bottle</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>25</td>
</tr>
<tr>
<td>8</td>
<td>23</td>
</tr>
<tr>
<td>9</td>
<td>21</td>
</tr>
<tr>
<td>10</td>
<td>19</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of Carnival Ride Tickets</th>
<th>Cost (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>32</td>
<td>30</td>
</tr>
</tbody>
</table>
Day 4: Multiple Representations of Linear Functions

**Standard: MGSE9–12.F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Linear functions can be represented in multiple ways.

<table>
<thead>
<tr>
<th>Set</th>
<th>Words</th>
<th>Algebra (equation)</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>(table, mapping, list)</td>
<td>Luigi’s plumbing service charges 30 dollars to make a house call plus 20 dollars per hour of service.</td>
<td>( L(h) = 20h + 30 )</td>
<td><img src="image" alt="Graph" /></td>
</tr>
</tbody>
</table>

For each of the following examples, determine the slope and y-intercept, write an equation in function notation, and evaluate the function for the given input.

**Scenario 1:** Bennett and his friends decide to go bowling. The cost for the group is $12 for shoe rentals plus $4.00 per game. How much will it cost to play 3 games?

**Scenario 2:** How much will the salesman make if he sells 8 cars?

<table>
<thead>
<tr>
<th>Cars Sold</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daily Pay</td>
<td>200</td>
<td>250</td>
<td>300</td>
<td>350</td>
<td>650</td>
</tr>
</tbody>
</table>
Scenario 3: The following function represents the cost of a tow service based on the number of miles the vehicle is towed: \[ T(m) = \frac{1}{4}m + 25. \] How much will it cost to tow a car 90 miles?

Scenario 4: How much will it cost to fill up a 16 gallon tank?

![Graph showing cost of gas and gallons](image)

Scenario 5: Consider the following scenario and answer the questions below.

You came home to find a pipe of yours has busted! You need to hire a plumber quickly, but also have a budget to consider!

<table>
<thead>
<tr>
<th>Paul's Plumbing</th>
<th>Peter's Pipers</th>
</tr>
</thead>
<tbody>
<tr>
<td>hours</td>
<td>charge</td>
</tr>
<tr>
<td>2</td>
<td>70</td>
</tr>
<tr>
<td>4</td>
<td>110</td>
</tr>
<tr>
<td>7</td>
<td>170</td>
</tr>
</tbody>
</table>

a. Which business charges more per hour?

b. Which charges more for the consultation?

c. What equations model both business' pricing based on their hours of labor?

d. If the job took 8 hours and you hired Peter, how much money did you save?
Scenario 6: Suppose you receive $100 for a graduation present, and you deposit it into a savings account. Then each week after that, you add $20 to your savings account. When will you have $460?

Scenario 7: A car owner recorded the number of gallons of gas remaining in the car's gas tank after driving several miles. Use the graph below to answer the following questions.

a. What is the slope/rate of change?

b. What does x-intercept represent on the graph?

c. What does the y-intercept represent on the graph?

d. What does the point (200, 12) represent on the graph?
Day 5: Graphing Quadratic Functions in Standard Form

The parent function of a function is the simplest form of a function.

The parent function for a quadratic function is \( y = x^2 \) or \( f(x) = x^2 \). Graph the parent function below.

<table>
<thead>
<tr>
<th>x</th>
<th>x^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

The U-shaped graph of a quadratic function is called a ____________.

The highest or lowest point on a parabola is called the ____________.

One other characteristic of a quadratic equation is that one of the terms is always ____________.

Standard Form of a Quadratic Function:

\[ y = ax^2 + bx + c \]

The ____________ is \( x = \frac{-b}{2a} \).

The ____________ is on the axis of symmetry line. Look for that x-value in your table.

The a-value determines whether your graph “goes up” on both sides or “goes down” on both sides of your vertex.

- ____________: a-value is positive (looks like a “U”)
- ____________: a-value is negative (looks like an “∩”)

The ____________ / ____________ / ____________ / ____________ are where \( y = 0 \).

You can either solve the equation \( 0 = ax^2 + bx + c \), to find the roots or look for where \( y = 0 \) in your table.

The ____________ is where \( x = 0 \). This will be the point \((0, c)\).

A good PARABOLA has at least five points. Make a table of values with your vertex in the middle and plot them to make a good graph.
Steps for Graphing in Standard Form

1) Find the vertex.
   • Use \( x = \frac{-b}{2a} \) to find our x-coordinate of our vertex
   • Substitute that x back into our equation, and our solution is the y-coordinate of our vertex.

2) Use your vertex as the center for your table and determine two x values to the left and right of your x-coordinate and substitute those x values back into the equation to determine the y values.

3) Plot your points and connect them from left to right! Your table MUST have 5 points!

Example: Graph \( y = -2x^2 - 4x + 6 \)

\[ a = -2 \quad b = -4 \quad c = 6 \]

\[
x = \frac{-b}{2a} = \frac{-(-4)}{2(-2)} = \frac{4}{-4} = -1
\]

\[
y = -2(-1)^2 - 4(-1) + 6 = 8
\]

This parabola has an \___________\ at \( x = -1 \), a \___________\ at \((-1, 8)\) which is also considered a \___________\, a \___________\ at \(0, 6\), and \___________\ at \((-3, 0)\) and \(1, 0)\).

Example 1: Graph \( y = x^2 - 2x - 3 \)

\[ a = \quad b = \quad c = \]

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td>6</td>
</tr>
<tr>
<td>-1</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Y-Intercept? 
X-Intercepts? 
Up or Down? 
Maximum or Minimum?
Example 2: Graph: $y = 3x^2 - 6x$.

- $a = \phantom{0} \quad b = \phantom{0} \quad c = \phantom{0}$
- Vertex? ( , )

Y-Intercept?
X-Intercepts?
Up or Down?
Maximum or Minimum?

Example 3: Graph $y = 2x^2 + 3$.

- $a = \phantom{0} \quad b = \phantom{0} \quad c = \phantom{0}$
- Vertex? ( , )

Y-Intercept?
X-Intercepts?
Up or Down?
Maximum or Minimum?

Example 4: Graph: $y = -x^2 + 6x - 9$

- $a = \phantom{0} \quad b = \phantom{0} \quad c = \phantom{0}$
- Vertex? ( , )

Y-Intercept?
X-Intercepts?
Up or Down?
Maximum or Minimum?
Day 6: Graphing Quadratic Functions (Vertex Form)

Standard(s): MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

**Vertex Form of a Quadratic Function**

\[ y = a (x - h)^2 + k \]

**Vertex:** \((h, k)\)

The \(_______________________\) is \(x = h\). (Opposite of \(h\))

The \(_______________\) is on the axis of symmetry line at \((h, k)\). \textbf{Remember: the sign of “h” is the opposite.}

The \(a\)-value determines whether your graph “goes up” on both sides or “goes down” on both sides of your vertex.

- \(___________\): \(a\)-value is positive (looks like a “U”)
- \(___________\): \(a\)-value is negative (looks like an “∩”)

A good **PARABOLA** has at least five points. Make a table of values with your vertex in the middle and plot them to make a good graph.

**Transformations**

- If the \(a\)-value is negative, your graph has been **REFLECTED** over the x-axis.
- If the \(a\)-value (ignoring the negative) is less than one, your graph has been **SHRUNK** or **COMPRESSED** vertically.
- If the \(a\)-value (ignoring the negative) is bigger than one, your graph has been **STRETCHED** vertically.
- The location of the vertex determines where the graph has been **SHIFTED** or **TRANSLATED**.
**Graphing in Vertex Form**

**Example 1:** Graph \( y = (x - 1)^2 - 2 \).

\[
\begin{align*}
& a = \quad h = \quad k = \\
& \text{Vertex} = (\quad , \quad )
\end{align*}
\]

Transformations?

Up or Down?  
Maximum or Minimum?

**Example 2:** Graph: \( y = -3(x + 4)^2 + 1 \).

\[
\begin{align*}
& a = \quad h = \quad k = \\
& \text{Vertex} = (\quad , \quad )
\end{align*}
\]

Transformations?

Up or Down?  
Maximum or Minimum?

**Example 3:** Graph \( y = 2x^2 + 3 \).

\[
\begin{align*}
& a = \quad h = \quad k = \\
& \text{Vertex} = (\quad , \quad )
\end{align*}
\]

Transformations?

Up or Down?  
Maximum or Minimum?
**Day 7: Graphing Quadratics in Intercept (Factored) Form**

**Standard(s): MGSE9-12.F.IF.7** Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

We learned in Unit 3A how to factor, but we can also graph in factored form!

**Factored Form of a Quadratic Function:**

\[ y = a(x - p)(x - q) \]

To find the x-coordinate of the VERTEX, use the formula: \[ x = \frac{p+q}{2} \]

The ROOTS/ZEROS/X-INTERCEPTS are (p, 0) and (q, 0).

The a-value determines whether your graph “goes up” on both sides or “goes down” on both sides of your vertex.

- **MINIMUM**: a-value is positive (looks like a “U”)
- **MAXIMUM**: a-value is negative (looks like an “∩”)

A good PARABOLA has at least five points. Make a table of values with your vertex in the middle and plot them to make a good graph.

**Graphing in Factored/Intercept Form**

1. Find the vertex.
   - Use the formula \[ x = \frac{p+q}{2} \] to find our x-coordinate of our vertex
   - Substitute that x back into our equation, and our solution is the y-coordinate of our vertex.
2. Determine your two x-intercepts.
3. Plot your points and connect them from left to right! Your table MUST include 5 points.

**Example:** Graph \( y = (x - 1)(x - 3) \)

Roots/x-intercepts: \( p = 1 \) and \( q = 3 \)

Axis of Symmetry:

\[ x = \frac{p + q}{2} = \frac{1 + 3}{2} = \frac{4}{2} = 2 \]

\[ y = (2 - 1)(2 - 3) = (1)(-1) = -1 \]

<table>
<thead>
<tr>
<th>( X )</th>
<th>( Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

This parabola has an _____________ at \( x = 2 \), a _____________ at \( 2, -1 \) which is also considered a _____________.
a _____________ at \( (0, 3) \), and _____________ at \( (1, 0) \) and \( (3, 0) \).
Graphing in Factored/Intercept Form

**Example 1:** Graph $y = (x + 2)(x - 2)$.

- X-intercepts:
  - 
  - 
  - 

- Axis of symmetry:
  - 
  - 
  - 

- Vertex:
  - 
  - 

- Up or Down?
  - 

- Maximum or Minimum?
  - 

**Example 2:** Graph $y = -(x + 1)(x - 7)$

- X-intercepts:
  - 
  - 
  - 

- Axis of symmetry:
  - 
  - 
  - 

- Vertex:
  - 
  - 

- Up or Down?
  - 

- Maximum or Minimum?
  - 

**Example 3:** Graph $y = 2(x - 1)(x - 3)$.

- X-intercepts:
  - 
  - 
  - 

- Axis of symmetry:
  - 
  - 
  - 

- Vertex:
  - 
  - 

- Up or Down?
  - 

- Maximum or Minimum?
Day 8: Converting Between Forms of a Parabola

<table>
<thead>
<tr>
<th>Vertex Form</th>
<th>Standard Form</th>
<th>Intercept Form (Factored Form)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = a(x-h)^2 + k$</td>
<td>$y = ax^2 + bx + c$</td>
<td>$y = a(x-p)(x-q)$</td>
</tr>
<tr>
<td>$(h, k)$ is the vertex</td>
<td>$c$ is the $y$-intercept</td>
<td>$p$ and $q$ are $x$-intercepts</td>
</tr>
</tbody>
</table>

$a$ always determines the way the graph opens

**Converting from Standard Form to Factored Form**

$y = ax^2 + bx + c$ \quad \rightarrow \quad y = a(x-p)(x-q)$

Determine your vertex $(h, k)$ and keep the same $a$-value.

**Convert the following from standard form to factored form.**

a) $f(x) = x^2 + 6x - 7$ 

b) $y = 4x^2 + 18x + 8$

**Converting from Standard Form to Vertex Form**

$y = ax^2 + bx + c$ \quad \rightarrow \quad y = a(x-h)^2 + k$

Determine your vertex $(h, k)$ and keep the same $a$-value. The $x$-coordinate of the vertex is $x = \frac{-b}{2a}$

**Convert the following from standard form to vertex form.**

a) $y = x^2 - 2x - 3$ 

b) $y = -2x^2 + 12x - 18$
Converting from Factored Form to Standard Form

\[ y = a(x - p)(x - q) \quad \rightarrow \quad y = ax^2 + bx + c \]

Multiply your expressions together and place in standard form. Distribute “a” value if necessary.

Convert the following from factored form to standard form and list the y-intercept.

a) \( y = 2(x - 1)(x - 3) \) 
b) \( f(x) = -(x - 3)^2 \)

Converting from Vertex Form to Standard Form

\[ y = a(x - h)^2 + k \quad \rightarrow \quad y = ax^2 + bx + c \]

Expand your squared binomial, multiply the binomials, and add constants. Distribute “a” value if necessary. **Don't forget to add the constant!!**

Convert the following from vertex form to standard form and list the y-intercept.

a) \( y = (x - 5)^2 - 12 \) 
b) \( y = -3(x + 1)^2 + 4 \)
To convert from vertex form standard form, we are only going to focus on the right side of the equation. Look at the following example from above, but this time, we are going from standard to vertex.

<table>
<thead>
<tr>
<th>Steps</th>
<th>Reasoning/Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = x^2 + 8x + 11$</td>
<td>Original Equation</td>
</tr>
<tr>
<td>$y = (x^2 + 8x) + 11$</td>
<td>When completing the square, we only want to consider the $x^2$ &amp; $x$ terms</td>
</tr>
<tr>
<td>$y = (x^2 + 8x + ___) + 11 - __$</td>
<td>Since we are only working on one side of the equation, we want to add and subtract whatever number allows us to “complete the square” so the function doesn’t change in value (we are technically adding zero).</td>
</tr>
<tr>
<td>$y = (x^2 + 8x + 4^2) + 11 - 4^2$</td>
<td>When completing the square, take half of the $b$-value and square it.</td>
</tr>
<tr>
<td>$y = (x + 4)^2 + 11 - 16$</td>
<td>We can rewrite the perfect square trinomial as a binomial square (essentially we factored $x^2 + 8x + 16$).</td>
</tr>
<tr>
<td>$y = (x + 4)^2 - 5$ Vertex: (-4, -5)</td>
<td>Combine the like terms (11 &amp; -16).</td>
</tr>
</tbody>
</table>

Finding Vertex Form by Completing the Square

Convert to vertex form of the quadratic functions by completing the square.

1) $f(x) = x^2 + 6x + 11$
2) $y = x^2 - 10x + 2$
3) \( y = 2x^2 - 12x + 16 \)

4) \( h(x) = -2x^2 + 8x - 4 \)

5) \( g(x) = -3x^2 + 24x - 41 \)

6) \( h(x) = 6x^2 - 84x + 290 \)
**Day 10: Applications of the Vertex**

**Standard(s): MGSE9-12.F.IF.4** Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities.

<table>
<thead>
<tr>
<th>Words that Indicate Finding Vertex</th>
<th>Quadratic Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum/Maximum</td>
<td>Standard Form: ( y = ax^2 + bx + c ) y-int: (0, c)</td>
</tr>
<tr>
<td>Minimize/Maximize</td>
<td>Vertex Form: ( y = a(x - h)^2 + k ) vertex: (h, k)</td>
</tr>
<tr>
<td>Least/Greatest</td>
<td>Factored Form: ( y = a(x - p)(x - q) ) x-int: (p, 0) &amp; (q, 0)</td>
</tr>
<tr>
<td>Smallest/Largest</td>
<td></td>
</tr>
</tbody>
</table>

**Quadratic Keywords**

- **At the vertex:**
  - **x-value:** When is the object at max height?
  - **y-value:** How high does the object get?

- **How high is the object after ____ seconds?**
  - Plug given seconds into the graph’s equation.

- **How high off the ground did the object start?**

- **Zero at the positive zero:**
  - When does the object hit the ground?
  - How long is the object in the air?

- **Zero at the vertex:**
  - This zero gets thrown out for being too negative!
**Scenario 1.** The arch of a bridge forms a parabola modeled by the function \( y = -0.2(x - 40)^2 + 25 \), where \( x \) is the horizontal distance (in feet) from the arch’s left end and \( y \) is the corresponding vertical distance (in feet) from the base of the arch. How tall is the arch?

**Scenario 2.** Suppose the flight of a launched bottle rocket can be modeled by the equation \( y = -x^2 + 6x \), where \( y \) measures the rocket’s height above the ground in meters and \( x \) represents the rocket’s horizontal distance in meters from the launching spot at \( x = 0 \).

a. How far has the bottle rocket traveled horizontally when it reaches its maximum height? What is the maximum height the bottle rocket reaches?

b. How far does the bottle rocket travel in the horizontal direction from launch to landing?

**Scenario 3.** A frog is about to hop from the bank of a creek. The path of the jump can be modeled by the equation \( h(x) = -x^2 + 4x + 1 \), where \( h(x) \) is the frog’s height above the water and \( x \) is the number of seconds since the frog jumped. A fly is cruising at a height of 5 feet above the water. Is it possible for the frog to catch the fly, given the equation of the frog’s jump?
Scenario 4. A baker has modeled the monthly operating costs for making wedding cakes by the function
\[ y = 0.5x^2 - 12x + 150, \]
where \( y \) is the total costs in dollars and \( x \) is the number of cakes prepared.

a. How many cakes should be prepared each month to yield the minimum operating cost?

b. What is the minimum monthly operating cost?

Falling Objects: 
\[ h = -16t^2 + h_0 \]
\( h_0 = \) starting height, \( h = \) ending height

Scenario 5. The tallest building in the USA is in Chicago, Illinois. It is 1450 ft tall. How long would it take a penny
to drop from the top of the building to the ground?

Scenario 6. When an object is dropped from a height of 72 feet, how long does it take the object to hit the
ground?