Name: _____ Block: _____ Teacher: _____

Algebra 1

Unit 4 Notes: Modeling and Analyzing Exponential Functions

DISCLAIMER: We will be using this note packet for Unit 4. You will be responsible for bringing this packet to class EVERYDAY. If you lose it, you will have to print another one yourself. An electronic copy of this packet can be found on my class blog.

Standards
MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from exponential functions (integer inputs only).
MGSE9-12.A.CED.2 Create exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase "in two or more variables" refers to formulas like the compound interest formula, in which $A = P(1 + r/n)^{nt}$ has multiple variables.)
Build a function that models a relationship between two quantities **MGSE9-12.F.BF.1** Write a function that describes a relationship between two quantities
MGSE9-12.F.BF.1a Determine an explicit expression and the recursive process (steps for calculation) from context. For example, if Jimmy starts out with \$15 and earns \$2 a day, the explicit expression "2x+15" can be described recursively (either in writing or verbally) as "to find out how much money Jimmy will have tomorrow, you add \$2 to his total today." $J_n = J_{n-1} + 2$, $J_0 = 15$
MGSE9-12.F.BF.2 Write geometric sequences recursively and explicitly, use them to model situations, and translate between the two forms. Connect geometric sequences to exponential functions.
Build new functions from existing functions **MGSE9-12.F.BF.3** Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. (Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.)
Understand the concept of a function and use function notation **MGSE9-12.F.IF.1** Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and f(x) is the output (an element of the range). Graphically, the graph is y = f(x).
MGSE9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MGSE9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. (Generally, the scope of high school math defines this subset as the set of natural numbers 1,2,3,4...) By graphing or calculating terms, students should be able to show how the recursive sequence $a_1=7$, $a_n=a_{n-1} +2$; the sequence $s_n = 2(n-1) + 7$; and the function f(x) = 2x + 5 (when x is a natural number) all define the same sequence.
Interpret functions that arise in applications in terms of the context **MGSE9-12.F.IF.4** Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior.
MGSE9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
MGSE9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
Analyze functions using different representations **MGSE9-12.F.IF.7** Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.
MGSE9-12.F.IF.7e Graph exponential functions, showing intercepts and end behavior.
MGSE9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.

Unit 4: Exponential Functions

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After completion of this unit, you will be able to…

Learning Target #1: Graphs of Exponential Functions

- Evaluate an exponential function
- Graph an exponential function using a xy chart

Learning Target #2: Applications of Exponential Functions
- Create an exponential growth and decay function
- Evaluate the growth/decay function
- Create a compound interest function
- Evaluate a compound interest function
- Solve an exponential equation

Learning Target #3: Sequences
- Create an arithmetic sequence
- Create a geometric sequence

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Timeline for Unit 4

Monday	Tuesday	Wednesday	Thursday	Friday
October 28th	**29th**	**30th** Day 1 – Graphing Exponential Functions	**31st** Day 2 – Applications of Exponentials (Growth & Decay)	**November 1st** Day 3 – Applications of Exponential Functions (Compound Interest)
4th Day 4 – Explicit Sequences – Geometric & Arithmetic	**5th** **No School - Teacher Work Day**	**6th** Day 5 – Recursive Sequences – Geometric & Arithmetic	**7th** Unit 4 Review	**8th** **Unit 4 Test**

Day 1 – Exponential Functions $y = ab^x$

Standard(s): MGSE9-12.A.CED.2
Create exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Exploring Exponential Functions
Which of the options below will make you the most money after 15 days?

a. Earning $1 a day?

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
y															

b. Earning a penny at the end of the first day, earning two pennies at the end of the second day, earning 4 pennies at the end of the third day, earning 8 pennies at the end of the fourth day, and so on?

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
y															

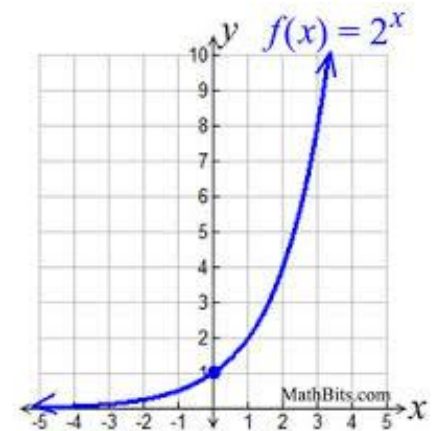The general form of an exponential function is:
y = ab^x

a represents your start/initial value/y-intercept
b represents your change

Features:
- Variable is in the exponent versus the base
- Start small and increase quickly or vice versa
- Asymptotes (graph heads towards a horizontal line but never touches it)
- Constant Ratios (multiply by same number every time)

Evaluating Exponential Functions

For exponential functions, the variable is in the exponent, but you still evaluate by plugging in the value given.

Example 1: Evaluate each exponential function.

a. $f(x) = 2(3)^x$ when x = 5   b. $y = 8(0.75)^x$ when x = 3   c. $f(x) = 4^x$, find f (2).

Graphing Exponential Functions

The general form of an exponential function is:

$$y = ab^x$$

Where **a** represents your starting or initial value
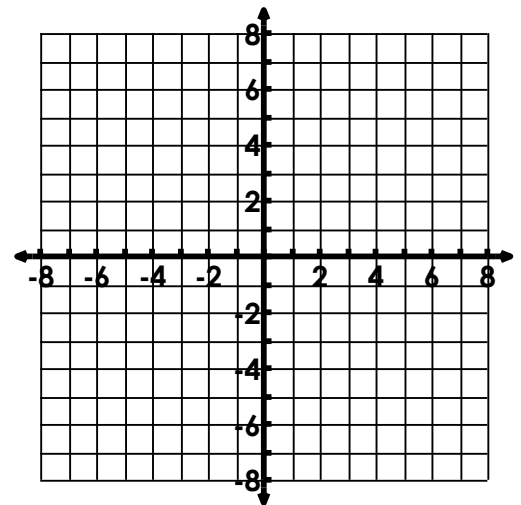**b** represents your growth/decay factor (change)

An **asymptote** is a line that an exponential graph gets closer and closer to but never touches or crosses.
The equation for the line of an asymptote is always y = _____.

Graph the following:

a. $y = 3(2)^x$

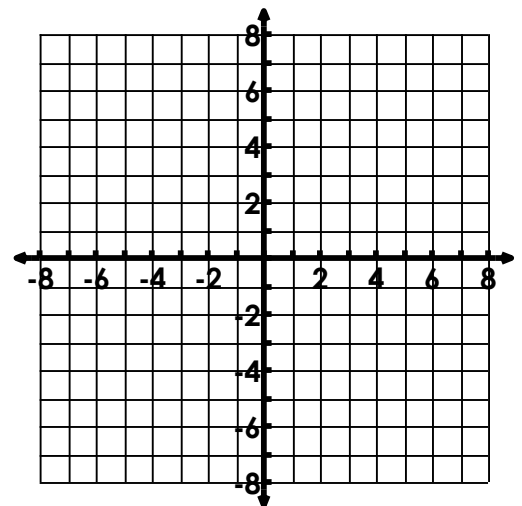| x	$y = 3(2)^x$
-2	
-1	
0	
1	
2	

Growth or decay?

Asymptote: _____

Y-intercept: _____

b. $y = 3\left(\dfrac{1}{2}\right)^x$

x	$y = 3\left(\dfrac{1}{2}\right)^x$
-2	
-1	
0	
1	
2	

Growth or decay?

Asymptote: _____

Y-intercept: _____

Creating Exponential Functions

Exponential Functions	a = Start/initial amount/y-int
$$y = ab^x$$	b = Change (growth/decay) x = How often change occurs y = Result of change over time

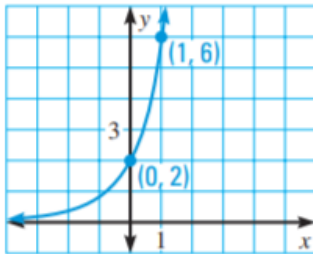**Write the equations that model these exponential functions.**
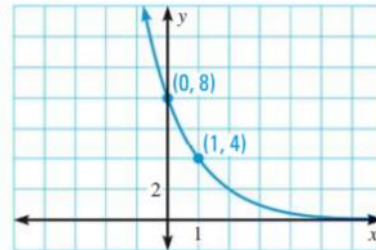
1.

x	−1	0	1	2
y	50	10	2	0.4

2.

Position, x	1	2	3	4	5
Term, y	128	64	32	16	8

3.

4.

5. March Madness is an example of exponential decay. At each round of the tournament, only the winning teams stay, so the number of teams playing at each round is half of the number of teams playing in the previous round. If 64 teams are a part of the official bracket at the start, how many teams are left after 5 rounds of play?

6. Bacteria have the ability to multiply at an alarming rate, where each bacteria splits into two new cells, doubling the number of bacteria present. If there are ten bacteria on your desk, and they double every hour, how many bacteria will be present tomorrow (desk uncleaned)?

7. Phosphorus-32 is used to study a plant's use of fertilizer. It has a half-life of 14 days. Write the exponential decay function for a 50-mg sample. Find the amount of phosphorus-32 remaining after 84 days.

6

8. **Which function represents this sequence?**

n	1	2	3	4	5	...
a_n	6	18	54	162	486	...

A. $f(n) = 3^{n-1}$
B. $f(n) = 6^{n-1}$
C. $f(n) = 3(6^{n-1})$
D. $f(n) = 6(3^{n-1})$

9. **The points (0, 1), (1, 5), (2, 25), and (3, 125) are on the graph of a function. Which equation represents that function?**

A. $f(x) = 2^x$
B. $f(x) = 3^x$
C. $f(x) = 4^x$
D. $f(x) = 5^x$

10. **The function graphed on this coordinate grid shows $f(x)$, the height of a dropped ball in feet after its xth bounce.**
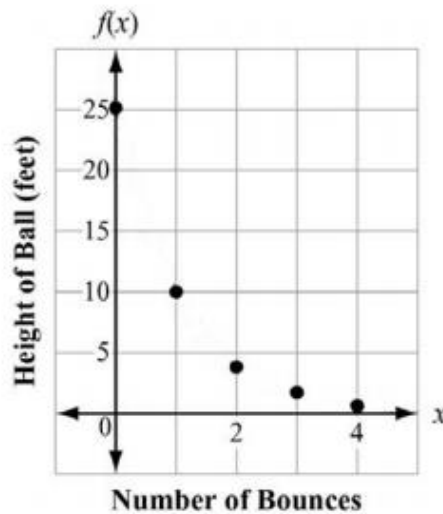
On which bounce was the height of the ball 10 feet?

A. bounce 1
B. bounce 2
C. bounce 3
D. bounce 4

Day 2 – Applications of Exponential Functions – Growth/Decay

Standard(s): MGSE9-12.A.CED.2
Create exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Review of Percentages: Remember percentages are always out of 100.

To change from a percent to a decimal:

Option 1: Divide by 100   **Option 2:** Move the decimal two places to the _____

25% = _____   6.5% = _____   2% = _____   10% = _____   3.05% = _____

Exponential Growth and Decay

Can you tell whether these functions represent growth or decay?

A. $y = 8(4)^x$ B. $f(x) = 2(5/7)^x$ C. $h(x) = 0.2(1.4)^x$ D. $y = ¾(0.99)^x$ E. $y = ½(1.01)^x$

When we discuss exponential growth and decay, we are going to use a slightly different equation than $y = ab^x$.

Exponential Growth	**Exponential Decay**
• A quantity increases over time	• A quantity decreases over time
$$y = a(1 + r)^t$$ where a>0	$$y = a(1 - r)^t$$ where a>0
y = final amount a = initial amount r = growth rate (express as decimal) t = time	y = final amount a = initial amount r = decay rate (express as decimal) t = time
(1 + r) represents the growth factor	(1 – r) represents the decay factor

Finding Growth and Decay Rates

Identify the following equations as growth or decay. Then identify the initial amount, growth/decay factor, and the growth/decay percent.

a. $y = 3.5(1.03)^t$

Growth/Decay: **growth**

Initial Amount: **3.5**

Growth/Decay Factor: **1.03**

Growth/Decay %: **0.03 = 3%**

b. $f(t) = 10,000(0.95)^t$

Growth/Decay: _____

Initial Amount: _____

Growth/Decay Factor: _____

Growth/Decay %: _____

c. $y = 2,500(1.2)^t$

Growth/Decay: _____

Initial Amount: _____

Growth/Decay Factor: _____

Growth/Decay %: _____

Growth and Decay Word Problems

Example 1: The original value of a painting is $1400 and the value increases by 9% each year. Write an exponential growth function to model this situation. Then find the value of the painting in 25 years.

Growth or Decay: _____

Starting value (a): _____

Rate (as a decimal): _____

Function: _____

Example 2: The cost of tuition at a college is $15,000 and is increasing at a rate of 6% per year. Find the cost of tuition after 4 years.

Growth or Decay: _____

Starting value (a): _____

Rate (as a decimal): _____

Function: _____

Example 3: The value of a car is $18,000 and is depreciating at a rate of 12% per year. How much will your car be worth after 10 years?

Growth or Decay: _____

Starting value (a): _____
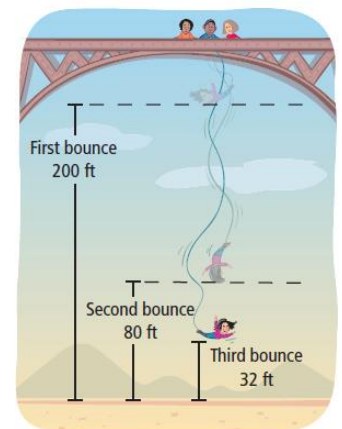
Rate (as a decimal): _____

Function: _____

Example 4: A bungee jumper jumps from a bridge. The diagram shows the bungee jumper's height above the ground at the top of each bounce. What is the bungee jumper's height at the top of the 5th bounce?

Growth or Decay: _____

Starting Value: _____

Rate (as a decimal): _____

Function: _____

First bounce
200 ft

Second bounce
80 ft

Third bounce
32 ft

Day 3 – Applications of Exponential Functions – Compound Interest

Standard(s): MGSE9-12.A.CED.2
Create exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

In middle school, you learned about **simple interest**, which is interest that is only earned on the original amount of money, called the principal. It's formula is *I = Prt,* where P represents principal, r represents rate, t represents time, and I represents interest.

Compound Interest is interest earned or paid on both the original amount (principal) and previously earned interest.

Compound Interest

$$A = P(1 + \frac{r}{n})^{nt}$$

A = balance after *t* years
P = Principal (original amount)
r = interest rate (as a decimal)
n = number of times interest is compounded per year
t = time (in years)

Example 1: Write a compound interest function that models an investment of $1000 at a rate of 3% compounded quarterly. Then find the balance after 5 years.

P = _____

r = _____

n = _____

t = _____

Example 2: Write a compound interest function that models an investment of $18,000 at a rate of 4.5% compounded annually. Then find the balance after 6 years.

P = _____

r = _____

n = _____

t = _____

Example 3: Write a compound interest function that models an investment of $4,000 at a rate of 2.5% compounded monthly. Then find the balance after 10 years.

P = _____

r = _____

n = _____

t = _____

Solving Exponential Equations

An **exponential equation** is an equation containing one or more expressions that have a variable as an exponent. When solving exponential equations, you want to rewrite the equations, so they have the same bases. If they have the same bases, you set the exponents equal to each other.

If $b^x = b^y$, then $x = y$

Solve: $2^{x-4} = 2^{-x+10}$	**Explanation**

Solve the following equations.

a. $3^{3x-7} = 3^{x+1}$

b. $\left(\dfrac{1}{2}\right)^x = \left(\dfrac{1}{2}\right)^{4x-12}$

c. $2^{2x-6} = 4$

d. $5^{4x-1} = 125$

Day 4 – Arithmetic and Geometric Sequences (Explicit Formulas)

> **Standard(s):**
> **MGSE9-12.F.BF.2** Write geometric sequences recursively and explicitly, use them to model situations, and translate between the two forms. Connect geometric sequences to exponential functions.
>
> **MGSE9-12.F.BF.1a** Determine an explicit expression and the recursive process (steps for calculation) from context.

A sequence is a list of numbers or objects, called **terms**, in a certain order.

Arithmetic Sequences
- The difference between any two _____ terms is always the same.
- This number is called a _____ _____
- The number is _____ or _____ to any term to achieve the next term.

Geometric Sequences
- The _____ of consecutive terms is always the same.
- This number is called the _____ _____.
- The number is _____ by any term to achieve the next term.

Identify the following sequences as arithmetic or geometric. Then name the common difference or common ratio.

1. 2, 6, 10, 14, …   2. 2, 6, 18, 54, …   3. 56, 84, 126, 189, …

4. 56, 26, -4, -34, …   7. 0.1, 1, 10, 100, …   8. 0.1, 0.15, 0.2, 0.25, …

Explicit Formulas

An explicit formula is a formula that allows you to find **any** term.

Arithmetic Sequence Explicit Formula $a_n = a_1 + d(n-1)$

Geometric Sequence Explicit Formula $a_n = a_1 \cdot r^{n-1}$

Example 1: Find the 21st term of the sequence 32, 26, 20, 14, 8, …

Example 2: Find the 11th term of the sequence 1024, 512, 256, …

Find the given term of each of the following sequences.

1. Given the sequence 25, 40, 55, 70, ... what is the 24th term?

2. Given the sequence 0.01, 0.2, 4, 80, ... what is the 9th term?

3. Given the sequence 88, 81, 74, 67, ... what is the 18th term?

4. Given the sequence 384, 96, 24, 6, ... what is the 7th term?

Practice

Use your knowledge of sequences to answer the following multiple-choice questions.

1. The formula of the *n*th term of the sequence $3, -6, 12, -24, 48 \ldots$ is

a. $a_n = -2(3)^n$ c. $a_n = -2(3)^{n-1}$

b. $a_n = 3(-2)^n$ d. $a_n = 3(-2)^{n-1}$

2. What is a formula for the *n*th term of sequence B shown ? $B = 10, 12, 14, 16, \ldots$

a. $b_n = 8 + 2n$ c. $b_n = 10(2)^n$

b. $b_n = 10 + 2n$ d. $b_n = 10(2)^{n-1}$

3. A sequence has the following terms: $a_1 = 4$, $a_2 = 10$, $a_3 = 25$, $a_4 = 62.5$. Which formula represents the *n*th term in the sequence?

a. $a_n = 4 + 2.5n$ c. $a_n = 4(2.5)^n$

b. $a_n = 4 + 2.5(n - 1)$ d. $a_n = 4(2.5)^{n-1}$

4. The third term in an arithmetic sequence is 10 and the fifth term is 26. If the first term is a₁, which is an equation for the *n*th term of this sequence?

a. $a_n = 8n + 10$ c. $a_n = 16n + 10$

b. $a_n = 8n - 14$ d. $a_n = 16n - 38$

Day 5 – Arithmetic and Geometric Sequences (Recursive Formulas)

Standard(s):
MGSE9-12.F.BF.2 Write geometric sequences recursively and explicitly, use them to model situations, and translate between the two forms. Connect geometric sequences to exponential functions.

MGSE9-12.F.BF.1a Determine an explicit expression and the recursive process (steps for calculation) from context.

A recursive formula is used to determine the **next** term based on the **previous** term.

Arithmetic Sequence Recursive Formula $\quad\quad \begin{cases} a_1 = \\ a_n = a_{n-1} + d \end{cases}$

Geometric Sequence Recursive Formula $\quad\quad \begin{cases} a_1 = \\ a_n = r \cdot a_{n-1} \end{cases}$

Match the following recursive formulas with their sequences.

1. 5, 15, 25, 35, …
2. 8, -20, 50, -125,…
3. 5, 15, 45, 135, …
4. 20, 17.5, 15, 12.5, …
5. -8, -3, 2, 7, …
6. 1000, 500, 250, 125, …
7. -99, -97, -95, -93, …
8. 2, 10, 50, 250, …

a. $a_n = a_{n-1} - 2.5$

b. $a_n = a_{n-1} + 2$

c. $a_n = a_{n-1} + 5$

d. $a_n = a_{n-1} + 10$

e. $a_n = 3a_{n-1}$

f. $a_n = 0.5a_{n-1}$

g. $a_n = 5a_{n-1}$

h. $a_n = -2.5a_{n-1}$

Write a recursive formula for the following sequences.

1. $3, -6, 12, -24, ...$
2. $3, -9, -21, -33, ...$

Use the given formulas to generate the first four terms of the corresponding sequences.

3. $\begin{cases} a_1 = 54 \\ a_n = \frac{1}{3} a_{n-1} \end{cases}$
4. $\begin{cases} a_1 = 10 \\ a_n = a_{n-1} + 3 \end{cases}$
5. $\begin{cases} a_1 = 10 \\ a_n = 3a_{n-1} \end{cases}$

Use your knowledge of sequences to answer the following multiple-choice questions.

1. The formula $\begin{cases} a_1 = 3000 \\ a_n = 0.80a_{n-1} \end{cases}$ can be used to model which scenario?

a. The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.

c. The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.

b. A bank account starts with a deposit of $3000, and each year it grows by 80%.

d. The initial value of a specialty toy is $3000, and its value each of the following years is 20% less.

2. At her job, Pat earns $25,000 the first year and receives a raise of $1000 each year. The explicit formula for the nth term of this sequence is $a_n = 25000 + 1000(n-1)$. Which rule best represents the equivalent recursive formula?

a. $a_1 = 25000; a_n = 1000a_{n-1}$

c. $a_1 = 25000; a_n = a_{n-1} + 1000$

b. $a_1 = 1000; a_n = 25000a_{n+1}$

d. $a_1 = 25000; a_n = a_{n+1} + 1000$

3. Which function represents this sequence?

n	1	2	3	4	5	...
a_n	-1	1	3	5	7	...

a. $a_n = a_{n-1} + 1$

c. $a_n = 2a_{n-1}$

b. $a_n = a_{n-1} + 2$

d. $a_n = 2a_{n-1} - 3$

4. A theater has more seats in the back rows than it has in the front rows. At a particular theater each row has two more seats than the row in front of it. Which formulas model this situation if the front row has twenty seats?
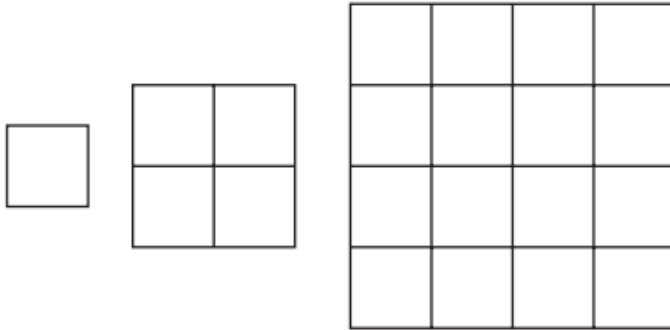
a. $a_n = a_{n-1} + 2$ and $a_n = 2n + 20$

c. $a_n = 2a_{n-1}$ and $a_n = 2n + 20$

b. $a_n = a_{n-1} + 2$ and $a_n = 2n + 18$

d. $a_n = 2a_{n-1}$ and $a_n = 2n + 18$

5. Select TWO of the following statements that are TRUE based on the following pictorial sequence.

Design 1   Design 2   Design 3   Design 4

a. $a_n = 2a_{n-1}$

d. $a_n = 3a_{n-1}$

b. $a_n = a_{n-1} + 2$

e. $a_n = 2n + 1$

c. $a_n = a_{n-1} - 2$

f. $a_n = 2n + 3$

1. Consider this pattern.

Which function represents the sequence that represents the pattern?

A. $a_n = (4)^{(n-1)}$

B. $a_n = (4)^{(a_n - 1)}$

C. $a_n = (a_n)(4)^{(n-1)}$

D. $a_n = (a_n)^4$

2. Consider this pattern.

Which function represents the sequence that represents the pattern?

A. $a_n = a_{n-1} - 3$
B. $a_n = a_{n-1} + 3$
C. $a_n = 3a_{n-1} - 3$
D. $a_n = 3a_{n-1} + 3$

3. Which explicit formula describes the pattern in this table?

d	c
0	1
1	6
2	36
3	216

A. $C = 6d$
B. $C = d + 6$
C. $C = 6^d$
D. $C = d^6$